On optimal stopping of risk reserve process

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Start

Abstract

The following problem in risk theory is considered. An insurance company, endowed with an initial capital $a \geq 0$, receives premiums and pays out claims that occur according to a renewal process $\{N(t), t \geq 0\}$. The times between consecutive claims are independent and identically distributed (i.i.d.). The sequence of successive claims is a sequence of i.i.d. random variables. The capital of the company is invested at interest rate $\alpha \in [0, 1]$, claims increase at rate $\beta \in [0, 1]$. The aim is to find the stopping time that maximizes the capital of the company. The cases of immediate claim payout as well as payout at the end of periods are considered.

Plan of presentation

- Introduction
- The optimization problem
- The solution of the optimization problem: finding the optimal stopping time when the number of claims is fixed
- The case with an infinite number of claims
- Final remarks

Introduction

The following problem in risk theory is considered. An insurance company, endowed with an initial capital $a \geq 0$, receives premiums and pays out claims that occur according to a renewal process $\{N(t), t \geq 0\}$, where $N(t)$ is the number of losses up till time t. Let T_i denote the time of the ith loss, then random variables $S_i = T_i - T_{i-1}$ are independent and identically distributed (i.i.d.) with cumulative distribution function (cdf) F; $T_0 = 0$. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with cdf H, which represents the discounted value of successive claims.

A typical process of capital assets of an insurance company is given by the formula

$$\bar{U}_t = a + ct - \sum_{n=0}^{N(t)} X_n, \tag{1}$$

where $a > 0$ is the initial capital, $c > 0$ is a constant rate of income from the insurance premiums, $X_0 = 0$ and $N(0) = 0$. Some interesting properties of the model given by (1) have been considered by E. Ferenstein & A. Sierociński [7]. U. Jensen [8] investigated a similar model with a claim process modulated by periodic Markovian processes. The models mentioned above do not take inflation into consideration, which is a serious omission.

We assume the initial capital of the insurance company and received premiums are invested at a constant rate of return $\alpha \in [0, 1]$. As a consequence of inflation, the value of claims increases at rate $\beta \in [0, 1]$. In this case the process of capital assets is

$$U_t = ae^{\alpha t} + \int_0^t ce^{\alpha(t-s)} ds - \sum_{n=0}^{N(t)} X_n e^{\beta T_n}, \tag{2}$$

A similar model with the claim process modulated by periodic Markovian processes has been investigated by Schöttl [11]. Schöttl has used a smooth semimartingale representation to solve the optimal stopping problem. We use dynamic programming to solve our model.

Models investigated by Muciek and Schöttl are still far away from the real economy. In fact both models were constructed assuming the claims are paid at the end of the investment period which in fact is not the case. The claims should be paid immediately and thus should decrease the invested capital.

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Such a situation is modeled by the following process

$$U_t = ae^{\alpha t} + \int_0^t ce^{\alpha(t-s)}ds - \sum_{n=0}^{N(t)} X_n e^{\beta T_n} e^{\alpha(t-T_n)}, \tag{3}$$

which reduces to

$$U_t = ae^{\alpha t} + \int_0^t ce^{\alpha(t-s)}ds - e^{\alpha t} \sum_{n=0}^{N(t)} X_n e^{\beta_1 T_n}, \tag{4}$$

where $\beta_1 = \beta - \alpha$. The equation (3) differs from (2) by the discount factor $e^{\alpha(t-T_n)}$. This change will have an impact on the form of the dynamic programming equations.

The return at time t is $\{Z(t), t \geq 0\}$:

$$Z(t) = \begin{cases} g_1(U_t)\mathrm{I}_{\{U_s>0,s\leq t\}} & \text{if } t \leq t_0, \\ 0 & \text{if } t > t_0, \end{cases} \qquad (5)$$

where g_1 is a utility function. A utility function is a subjective sense of one's wealth, commonly used in actuarial mathematics. Properties of utility functions are explained in detail in [3]. The return $Z(t)$ defined by (5) is positive ($Z(t) > 0$), only if the capital assets process has been positive all the time ($U_s > 0$ for all $s \leq t$) and not later than some fixed time t_0.

For simplicity of notation, let $g(u,t) = g_1(u)\mathrm{I}_{\{t \geq 0\}}$. Then

$$Z(t) = g(U_t, t_0 - t) \prod_{j=0}^{N(t)} \mathrm{I}_{\{U_{T_j} > 0\}}.$$

$Z(t)$ is then nonzero, if bankruptcy does not occur before time t, and if $t < t_0$.

The optimization problem

Let

$$\mathcal{F}(t) = \sigma(U_s, s \leq t) = \sigma(X_1, T_1, \ldots, X_{N(t)}, T_{N(t)})$$

be the σ-field generated by all the events up to time $t \geq 0$. First we fix the number of claims that may occur in our model (K). Let $\mathcal{T}$ be the set of all stopping times with respect to the family $\{\mathcal{F}(t), t \geq 0\}$. Furthermore, for $n = 0, 1, \ldots, k < K$ let $\mathcal{T}_{n,K}$ denote the subset of $\mathcal{T}$, such that

$$\tau \in \mathcal{T}_{n,K}, \quad \text{if and only if} \quad T_n \leq \tau \leq T_K \quad \text{a.s.}$$

Let $\mathcal{F}_n = \mathcal{F}(T_n)$ (note, the sequence $\mathcal{F}(T_n)$ is ascending). The essence of the considerations on the next slides will be to find the optimal stopping time τ_K^*, such that

$$\mathsf{E}Z(\tau_K^*) = \sup\{\mathsf{E}Z(\tau) : \tau \in \mathcal{T}_{0,K}\}. \tag{6}$$

In order to find the optimal stopping time τ_K^*, we first consider optimal stopping times $\tau_{n,K}^*$, such that

$$\mathsf{E}(Z(\tau_{n,K}^*)|\mathcal{F}_n) = \operatorname{ess\,sup}\{\mathsf{E}(Z(\tau)|\mathcal{F}_n) : \tau \in \mathcal{T}_{n,K}\} \tag{7}$$

and using backward induction as in dynamic programming, we will obtain $\tau_K^* = \tau_{0,K}^*$

After finding the optimal stopping time τ_K^* for fixed K we will deal with unlimited number of claims and the aim will be to find the optimal stopping time τ^*, such that

$$\mathsf{E}Z(\tau^*) = \sup\{\mathsf{E}Z(\tau) : \tau \in \mathcal{T}\} \tag{8}$$

is fulfilled. It will be shown that τ^* can be defined as the limit of the finite horizon optimal stopping times.

Optimal stopping time when the number of claims is fixed

The following representation lemma (see for example [5]) plays the crucial role in consequent considerations:

Lemma 1. *If* $\tau \in \mathcal{T}_{n,K}$*, there exists a positive,* $\mathcal{F}_n$*-measurable random variable* ξ*, such that*

$$\tau \wedge T_{n+1} = (T_n + \xi) \wedge T_{n+1} \quad \textit{a.s.}$$

The dynamic programming equations must satisfy

$$\Gamma_{n,K} = \mathsf{ess}\sup\{\mathsf{E}(Z(\tau)|\mathcal{F}_n) : \tau \in \mathcal{T}_{n,K}\}, \qquad n = K, K-1, \ldots, 1.$$

Let

$$\mu_n = \prod_{j=1}^{n} \mathrm{I}_{\{U_{T_j}>0\}}, \qquad \mu_0 = 1.$$

Then

$$\Gamma_{K,K} = Z(T_K) = g(U_{T_K}, t_0 - T_K)\mu_K.$$

The following theorem presents the dynamic programming equations:

Theorem 2.

(i) *For* $n = K-1, K-2, \ldots, 0$,

$$\Gamma_{n,K} = \operatorname{ess\,sup} \Big\{ \mu_n \bar{F}(\xi) g\left(U_{T_n} + \left(a + \tfrac{c}{\alpha}\right)\left(e^{\alpha(T_n+\xi)} - e^{\alpha T_n}\right), t_0 - T_n - \xi\right) + \\ + \mathsf{E}(\mathrm{I}_{\{\xi \geq S_{n+1}\}} \Gamma_{n+1,K} | \mathcal{F}_n) : \ \xi \geq 0 \text{ is } \mathcal{F}_n\text{-measurable} \Big\} \quad \text{a.s.},$$

where $\bar{F} = 1 - F$ *is the survival function.*

(ii) *For* $n = K, K-1, \ldots, 0$,

$$\Gamma_{n,K} = \mu_n \gamma_{K-n}(U_{T_n}, T_n) \quad \text{a.s.},$$

where the sequence of functions $\{\gamma_j(u,t), u \in \mathrm{R}, t \geq 0\}$ *is defined as follows*

$$\begin{aligned}
\gamma_0(u,t) &= g(u, t_0 - t), \\
\gamma_j(u,t) &= \sup_{r \geq 0} \Bigg[\bar{F}(r) g \left(u + \left(a + \frac{c}{\alpha} \right) \left(e^{\alpha(t+r)} - e^{\alpha t} \right), t_0 - t - r \right) + \\
&\quad + \int_0^r dF(s) \int_0^{u + \left(a + \frac{c}{\alpha}\right)\left(e^{\alpha(t+s)} - e^{\alpha t}\right)} \\
&\quad \gamma_{j-1} \left(u + \left(a + \frac{c}{\alpha} \right) \left(e^{\alpha(t+s)} - e^{\alpha t} \right) - x e^{\beta(t+s)}, t + s \right) dH(x) \Bigg], \\
&\qquad j = 1, 2, \ldots
\end{aligned}$$

The next step is to find the optimal stopping time τ_K^*. To do this we should analyze the properties of the sequence of functions $\{\gamma_n, n \geq 0\}$ which was defined in second part of the Theorem 2.

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In order to do this, we define some functions and operators, whose properties will be investigated.

Let $B = B[(-\infty, +\infty) \times [0, +\infty)]$ be the space of all bounded and continuous functions with the norm $||\delta|| = \sup_{u,t} |\delta(u,t)|$ and let

$$B^0 = \{\delta : \ \delta(u,t) = \delta_1(u,t)\mathrm{I}_{\{t \le t_0\}} \text{ and } \delta_1 \in B\}.$$

One should notice that the functions $\{\gamma_n, n \ge 0\}$ from Theorem 2 are included in B^0.

For each $\delta \in B^0$ and any $u \in \mathrm{R}$, $t, r \geq 0$ let

$$\begin{aligned}
\phi_\delta(r,u,t) \quad = \quad & \bar{F}(r) g\left(u + \left(a + \frac{c}{\alpha}\right)\left(e^{\alpha(t+r)} - e^{\alpha t}\right), t_0 - t - r\right) + \\
& + \int_0^r dF(s) \Bigg[\int_0^{u + \left(a+\frac{c}{\alpha}\right)\left(e^{\alpha(t+s)} - e^{\alpha t}\right)} \\
& \quad \delta\left(u + \left(a + \frac{c}{\alpha}\right)\left(e^{\alpha(t+s)} - e^{\alpha t}\right) - x e^{\beta(t+s)}, t + s\right) dH(x) \Bigg],
\end{aligned}$$

which in fact is the term under $\sup$ in the definition of $\{\gamma_n, n \geq 0\}$.

From the properties of the cumulative distribution function F we know that $\phi_\delta(r,u,t)$ has at most a countable number of points of discontinuity according to r and is continuous according to (u,t) in the case of $g_1(\cdot)$ being continuous and $t \neq t_0 - r$. Therefore, for further considerations we need the following assumptions

Assumption 3. *The function $g_1(\cdot)$ is bounded and continuous.*

Such an assumption is not a real restriction, since most commonly used utility functions meet these requirements.

For each $\delta \in B^0$ let

$$(\Phi\delta)(u,t) = \sup_{r \geq 0}\{\phi_\delta(r,u,t)\}. \tag{9}$$

Lemma 4. *For each $\delta \in B^0$ we have*

$$(\Phi\delta)(u,t) = \max_{0 \leq r \leq t_0 - t}\{\phi_\delta(r,u,t)\} \in B^0$$

and there exists a function $r_\delta(u,t)$ such that $(\Phi\delta)(u,t) = \phi_\delta(r_\delta(u,t),u,t)$.

For $i = 1, 2, \ldots$ and $u \in \mathrm{R}$, $t \geq 0$, $\gamma_i(u,t)$ may be expressed as follows

$$\gamma_i(u,t) = \begin{cases} (\Phi\gamma_{i-1})(u,t) & \text{if } u \geq 0 \text{ and } t \leq t_0, \\ 0 & \text{otherwise,} \end{cases}$$

and from Lemma 4 there exist functions $r_{\gamma_{i-1}}$ such that

$$\gamma_i(u,t) = \begin{cases} \phi_{\gamma_{i-1}}(r_{\gamma_{i-1}}, u, t) & \text{if } u \geq 0 \text{ and } t \leq t_0, \\ 0 & \text{otherwise.} \end{cases}$$

To specify the form of the optimal stopping times $\tau^*_{n,K}$, we need to define the following random variables

$$R^*_i = r_{\gamma_{K-i+1}}(U_{T_i}, T_i)$$

and

$$\sigma_{n,K} = K \wedge \inf\{i \geq n : \; R^*_i < S_{i+1}\}.$$

Finally in Corollary 5 we present the form of the optimal stopping time.

Corollary 5. *Let*

$$\tau^*_{n,K} = T_{\sigma_{n,K}} + R^*_{\sigma_{n,K}} \quad \textit{and} \quad \tau^*_K = \tau^*_{0,K},$$

then for all $0 \leq n \leq K$ *the following hold*

$$\Gamma_{n,K} = \mathsf{E}(Z(\tau^*_{n,K})|\mathcal{F}_n) \ \textit{a.s.} \quad \textit{and} \quad \Gamma_{0,K} = \mathsf{E}(Z(\tau^*_K)) = \gamma_K(a,0),$$

which means $\tau^*_{n,K}$ *and* τ^*_K *are optimal stopping times in the classes* $\mathcal{T}_{n,K}$ *and* $\mathcal{T}_{0,K}$ *respectively.*

The case with an infinite number of claims

While $\mathcal{T}$ is the set of all stopping times with respect to the family $\{\mathcal{F}(t), t \geq 0\}$, we would like to maximize the mean return (5), i.e. to find the optimal stopping time τ^*, such that

$$\mathsf{E}Z(\tau^*) = \sup\{\mathsf{E}Z(\tau) : \tau \in \mathcal{T}\} \tag{10}$$

is fulfilled. In fact τ^* can be defined as the limit of the finite horizon optimal stopping times.

Let τ_n^* be an optimal stopping time taken from the set of stopping times which occur not earlier then the time of n-th claim, T_n,

$$\mathsf{E}Z(\tau_n^*) = \sup\{\mathsf{E}Z(\tau) : \tau \in \mathcal{T} \cap \{\tau : \tau \geq T_n\}\}. \tag{11}$$

The solution of this case will be based on the iteration of the operator Φ defined by (9). By an assumption that the interarrival time is greater than t_0 with nonzero probability we will prove that the operator Φ is a contraction and it has a fixed point.

Lemma 6. *If $F(t_0) < 1$ then*

(i) *the operator $\Phi : B^0 \to B^0$ defined by (9) is a contraction.*

(ii) *there exists $\gamma \in B^0$, such that*

$$\gamma = \Phi\gamma \qquad \textit{and} \qquad \lim_{K\to\infty} ||\gamma_K - \gamma|| = 0.$$

The essence of this part is contained in the following theorem. The proof will be based on the proof of the existence of optimal stopping times for semi-Markov processes presented by Boshuizen & Gouweleeuw [2].

Theorem 7. *Assuming the utility function g_1 is differentiable and nondecreasing and F has a density function f we have*

(i) *for $n = 0, 1, \ldots$, the limit $\tilde{\tau}_n := \lim_{K\to\infty} \tau^*_{n,K}$ exists and $\tilde{\tau}_n$ is an optimal stopping time in $\mathcal{T} \cap \{\tau : \ \tau \geq T_n\}$ ($\tau^*_{n,K}$ is a solution of the case with finite number of claims defined by (7)),*

(ii) $\mathsf{E}[Z(\tilde{\tau}_n)|\mathcal{F}_n] = \mu_n \gamma(U_{T_n}, T_n)$ *a.s.*

P r o o f. (i) Let $n \geq 0$. As $\tau^*_{n,K} \leq \tau^*_{n,K+1}$ a.s., the stopping rule $\tilde{\tau}_n = \lim_{K\to\infty} \tau^*_{n,K} \geq T_n$ exists.

The optimality of $\tilde{\tau}_n$ will be proved in a similar way as in Boshuizen & Gouweleeuw [2]. Let $\xi_t = (t, U_t, Y_t, V_t)$, $Y_t = t - T_{N(t)}$, $V_t = \mu_{N(t)}$, $t \geq 0$. Then $\xi = \{\xi_t : t \geq 0\}$ is a Markov process with state space $\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}_+ \times \{0, 1\}$. The return $Z(t)$ is a function of ξ_t (say $\tilde{g}$). Let A be a strong generator of ξ. Basing on Theorem 11.2.2 from Rolski et al. [10] we get

$$(A\tilde{g})(t, u, y, v) = \left\{ e^{\alpha t}(a\alpha + c)g_1'(u) - \frac{f(y)}{\bar{F}(y)} \left[g_1(u) - \int_0^u g_1(u - e^{\beta t}x)dH(x) \right] \right\} v,$$

where $t < t_0$, $y \geq 0$ and $v \in \{0, 1\}$.

Note that $\tilde{g}(\xi_t) - \tilde{g}(\xi_0) - \int_0^t (A\tilde{g})(\xi_s)ds, t \geq 0$ is a martingale with respect to σ-$(\xi_s, s \leq t)$, which is the same as $\mathcal{F}(t)$ (see Rolski et al. [10], p. 442 and Davis [4], p. 31). Applying the optional sampling theorem ([4], p. 22) we get

$$\mathsf{E}[\tilde{g}(\xi_{\tau^*_{n,K}})|\xi_{T_n}] - \tilde{g}(\xi_{T_n}) = \mathsf{E}\left[\int_{T_n}^{\tau^*_{n,K}} (A\tilde{g})(\xi_s)ds \,\middle|\, \mathcal{F}_n\right] \quad \text{a.s.} \quad (12)$$

Since

$$(A\tilde{g})(\xi_s) = \left\{ e^{\alpha t}(a\alpha + c)g_1'(U_s) + \frac{f(s - T_{N(s)})}{\bar{F}(s - T_{N(s)})}\left[\int_0^{U_s} g_1(U_s - e^{\beta t}x)dH(x) - g_1(U_s)\right]\right\} \mu_{N(s)}$$

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we have

$$\mathsf{E}\left[\int_{T_n}^{\tau^*_{n,K}} (A\tilde{g})(\xi_s)ds|\mathcal{F}_n\right] = \mathsf{E}(I^1_{n,K}|\mathcal{F}_n) - \mathsf{E}(I^2_{n,K}|\mathcal{F}_n),$$

where

$$I^2_{n,K} = \int_{T_n}^{\tau^*_{n,K}} \frac{f(s - T_{N(s)})}{\bar{F}(s - T_{N(s)})} g_1(U_s)\mu_{N(s)}ds.$$

$I^1_{n,K}$ and $I^2_{n,K}$ are positive random variables and $I^2_{n,K}$ is bounded by

$$g_1\left(\left(a + \frac{c}{\alpha}\right)e^{\alpha t_0} - \frac{c}{\alpha}\right)\frac{\mathsf{E}(L)}{\bar{F}(t_0)},$$

where $L = \inf\{n \in \mathrm{N} : T_n < t_0, T_{n+1} \geq t_0\}$. Note that

$$\mathsf{E}(L) = \sum_{n=1}^{\infty} F^{*(n)}(t_0) \leq \sum_{n=1}^{\infty} [F(t_0)]^n < \infty.$$

Hence, from the convergence of $\tau^*_{n,K}$ to $\tilde{\tau_n}$ as $K \to \infty$ and from the Monotone Convergence Theorem, the right side of (12) converges to

$$\mathsf{E}\left[\int_{T_n}^{\tilde{\tau}_n} (A\tilde{g})(\xi_s) ds \,\middle|\, \mathcal{F}_n \right].$$

Since $\tilde{\tau}_n < \infty$ a.s., applying Dynkin's formula we get

$$\mathsf{E}\left[\int_{T_n}^{\tilde{\tau}_n} (A\tilde{g})(\xi_s)ds \,\middle|\, \mathcal{F}_n\right] = \mathsf{E}[\tilde{g}(\xi_{\tilde{\tau}_n})|\mathcal{F}_n] - \tilde{g}(\xi_{T_n}) \quad \text{a.s.}$$

So we have

$$\mathsf{E}[\tilde{g}(\xi_{\tau^*_{n,K}})|\mathcal{F}_n] \xrightarrow{K\to\infty} \mathsf{E}[\tilde{g}(\xi_{\tilde{\tau}_n})|\mathcal{F}_n] \quad \text{a.s.} \tag{13}$$

Now we will prove that $\tilde{\tau}_n$ is optimal in the class $\mathcal{T} \cap \{\tau : \tau \geq T_n\}$. Let τ be any stopping time from $\mathcal{T} \cap \{\tau : \tau \geq T_n\}$. Then, as $\tau^*_{n,K}$ is optimal in $\mathcal{T}_{n,K}$, we have for any K, that

$$\mathsf{E}[\tilde{g}(\xi_{\tau^*_{n,K}})|\mathcal{F}_n] \geq \mathsf{E}[\tilde{g}(\xi_{\tau \wedge T_K})|\mathcal{F}_n] \quad \text{a.s.}$$

Hence, a reasoning similar to that which leads to (13) gives

$$\mathsf{E}[\tilde{g}(\xi_{\tilde{\tau}_n})|\mathcal{F}_n] \geq \mathsf{E}[\tilde{g}(\xi_{\tau})|\mathcal{F}_n] \quad \text{a.s..} \tag{14}$$

This completes the proof of (i).

(ii) $\mathsf{E}[\tilde{g}(\xi_{\tau^*_{n,K}})|\mathcal{F}_n] = \mu_n \gamma_{K-n}(U_{T_n}, T_n)$ from Theorem 2. Lemma 6 and (14) give

$$\mathsf{E}[\tilde{g}(\xi_{\tau^*_{n,K}})|\mathcal{F}_n] \xrightarrow{K\to\infty} \mathsf{E}[\tilde{g}(\xi_{\tau})|\mathcal{F}_n] = \mu_n \gamma(U_{T_n}, T_n) \quad \text{a.s.}$$

This completes the proof of Theorem 7.

In the case with immediate payouts (3) the strong generator has the following form:

$$(A\tilde{g})(t,u,y,v) = \left\{ \left((c+1)e^{\alpha t} + \alpha u - c \right) g_1'(u) - \frac{f(y)}{\bar{F}(y)} \left[g_1(u) - \int_0^u g_1(u - e^{\beta_1 t} x) dH(x) \right] \right\} v,$$

where $t < t_0$, $y \geq 0$ and $v \in \{0, 1\}$.

$$(A\tilde{g})(\xi_s) = \left\{ e^{\alpha t}(a\alpha + c - \alpha \sum_{n=0}^{N(s)} X_n e^{\beta_1 T_n}) g_1'(U_s) + \frac{f(s - T_{N(s)})}{\bar{F}(s - T_{N(s)})} \left[\int_0^{U_s} g_1(U_s - e^{\beta_1 t} x) dH(x) - g_1(U_s) \right] \right\} \mu_{N(s)}.$$

It should also be marked that the limit of optimal stopping times as $K \to \infty$ coincide with the overall optimal stopping time. In fact we have

$$\bigcup_{K=1}^{\infty} \mathcal{T}_K = \mathcal{T},$$

From the form of the return $Z(t)$ defined in (5), the reward function is zero for $t > t_0$, thus with probability one the limit of optimal stopping times is finite and $\tau^* = \tilde{\tau}_0$.

Final remarks

One should observe that as α and β tend to 0 ($\alpha \to 0$, $\beta \to 0$), we get the solution of the economically static model found by Ferenstein & Sierociski [7].

In fact it holds that U_t in (2) equals $\bar{U}_t$ in (1) a.s. for $\alpha = \beta = 0$, so in this case models are identical. Thus we have

$$\lim_{\alpha \to 0} \left[\left(a + \frac{c}{\alpha} \right) \left(e^{\alpha(t+s)} - e^{\alpha t} \right) \right] = cs$$

and

$$x e^{\beta(t+s)} = x \qquad \text{for } \beta = 0.$$

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So Equation (9) for $\phi_\delta(r, u, t)$, which is crucial for determining the optimal stopping time with fixed K, reduces to Equation (17) in [7] and the solutions for a fixed and finite number of claims are the same. The reasoning concerning the case with an infinite number of claims leads to similar results. It means that optimal stopping times are the same in both models when $\alpha \to 0$ and $\beta \to 0$ a.s.

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Thank you

for your attention